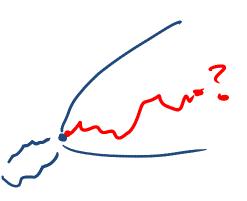


Risk Management and Governance The Notion of Risk

Prof. Hugues Pirotte

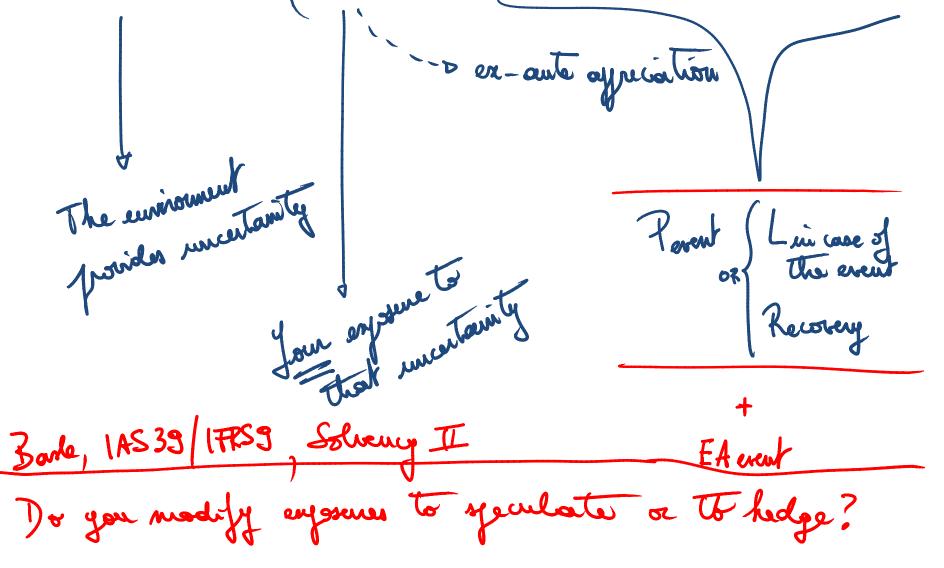


Peter Benertein Against the Gods: the history of rick.

UL B



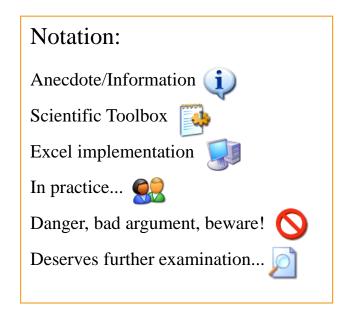
Uncertainty risk and components of risk



Risks interconnectedness IR noth FX note IRT Market votes. (Steh) Markets with (a wilder judenti Creeket Router (counterparty with Genational with System, Jeagle Inflation with (MACRO) joy) dunce not



Some notation during the course...



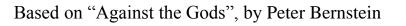
FIRUS D





Priors (Evolution)

- The (true) Greeks... 1.
- Gambling vs. Strategic management 🖛 2.
- Probabilists 3.
- Statistical sampling -
- Human behaviour
- Past, present & future and the measurement of uncertainty
 - \rightarrow "expecting the unexpected"
- 7. Allowing for vagueness
- 8. Measurement
- Regression to the mean 9.
- Management...what do we do? 10.
- Measuring ignorance 11.
- 12. Modelling and pretending humans behave like the model... Model Ruh





Priors (Old origins)

- Quant bases: 1654, 1700, 1875
 - » 1200-1300: Hindu-Arabic numbering system
 - » 1654: Blaise Pascal and Pierre de Fermat
 - ✓ Probability theory
 - » 1703: Jacob Bernoulli
 - ✓ Law of Large Numbers
 - » 1730: Abraham de Moivre
 - ✓ « Bell curve »
 - ✓ Law of Averages
 - » 1754: Thomas Bayes
 - Conditional probability theory
 - » 1875: Francis Galton
 - ✓ Regression to the mean

Based on "Against the Gods", by Peter Bernstein



Priors ("Recent" Dates and Facts)

- Some dates:
 - » 1857: 1 tuis "anun y factor" vare jullished.

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- » 1920: highly bullish market
- » 1929: Crisis
- October 1974: worst bear market since 1929
 Together with the OPEC, after adjustment for inflation, the entire rise from 1954 was erased!
 Bond market suffers from a 35% loss of purchasing power.
 Many of the go-go managers of the '60s disappeared along with Nixon's price controls.
- » 1987 & 1999-2002 : Brachy Commission & report
- » 2007-2008-...
- Some facts:
 - » 1969: Capital markets (S+B+Cash) = \$2'000 billion 1990: → \$ 22'000 billion
 - » 1950: 15% of US shares belong to pension funds 1992: → + than 50% (and 80% of trading activity!)
 - » 1970: 20% of US shares change hands within 1 year 1992: → + than 70%
 - » 1992: the average daily transaction volume = 100 times that of 1950!

1



Priors (Contemporaneous Finance History)

- The story begins with...
 - » Bachelier (1900): Theory of speculation & stochastic processes
 - "…opposite and totally diverging beliefs on expected market moves make buyers and sellers take opposite directions…"
 - "The mathematical expectation of a speculator is zero"
 - ✓ Rediscovered in the '50s by Jimmie Savage



Priors (Contemporaneous Finance History)

- » Financial news
 - ✓ Written on carbon before mid-1880
 - ✓ Introduction of small printing machines mid-1880
 - ✓ Charles Dow co-founder de Dow, Jones & Co in 1882 and first editor of Wall Street Journal (founded in 1885 by conversion of the Afternoon News Letter) → « the Dow theory »
 - « trends tend to be persistent until the market sends signals showing that these trends are losing their momentum and will reverse.
 - Dow Jones Average (1884): 9 railroads and 2 industrials
 - Publication of the first securities list made of pure industrials for the DJA
 - The single remaining company from the beginning: General Electric
 - ✓ Hamilton take the WSJ editorial board in 1903.
 - Continues to believe, following Dow, that prices can convey info about future prices, in opposition to Bachelier (! In 1960, groups meet.....and develop the theory of Market Efficiency)
 - October 21st 1929, Hamilton predicts the end of the bull market, just before he dies...
 - ...a fall of 90% of the market happens two days after.
 - Cascading bear market from 1931 to 1932.

1



Priors (Contemporaneous Finance History)

- » Cowles
 - ✓ Asks to a mathematician to run a regression with 20 variables → a revolution in terms of complexity
 - ✓ Data: <u>7'500 recommendations</u> from financial services, 4 years of transactions of insurance companies, 255 Hamilton editorials from 1903 to 1929 and 3'300 recommendations from financial publications
 - ✓ 6 on 16 financial services have done better and even in those cases, it is still difficult to explain their result by any other reason but luck!
 - Foundation of the Cowles Commission financing the Econometry Society which would publish Econometrica.
 - ✓ 1913: first publication of an index to become the famous S&P500 covering 97% of the 1933 market cap.
 - The goal of this index is to demonstrate what an investor would incur if it would have invested in every single stock of the NYSE at the beginning of 1871.
 - Cowles' Conclusion: « even if I did my negative surveys every five years, or others continued when I'm gone, it wouldn't matter. People are still going to subscribe to these services. They want to believe that somebody really knows. A world in which nobody really knows can be frightening. » (extract from Capital Ideas de Bernstein)

 (\mathbf{i})



Priors (Contemporaneous Finance History)

- » In 1952, Markowitz published an article in the JF : « Portfolio Selection »
 - \checkmark This subject starts to be discussed only in the 60s.
 - But a lot of publications were running on securities analysis and on the gold price.
 - Was sent to the Cowles Commission, but the economists there forwarded him to the business school. Reading Williams' work (DDM), he realizes that...
 - …investors have a real desire of diversification and that somewhere, the RISK dimension is as important as the RETURN dimension <> Keynes
 - ✓ Idea of the « Efficient Frontier! »
 - Markowitz receives his thesis even though Milton Friedman does not agree it to be on the economic ground, neither maths nor management. It was the first time that finance was considered as a truly research domain.
- » And then, the rush to analyze investors' individual preferences begins...
- » In 1958, Tobin wrote « Liquidity Preference as Behavior Toward Risk ». Leads to the separation theorem.

✓ This will drive us to the most modern version of portfolio theory...





Uncertainty: 1952–1973, The Golden Years

- 1952: Harry Markowitz* (EQ)
 - » Portfolio selection in a mean –variance framework
- 1953: Kenneth Arrow* / 1959: Debreu (avec Arrow) (EQ)
 - » Complete markets and the law of one price
 - » Allocation optimale des ressources en situation d'incertitude
- 1958 (et 1963): Franco Modigliani* and Merton Miller* (ARB)
 - » Value of company independant of financial structure
- 1961: Miller et Modigliani (ARB)
 - » Politique de dividendes
- 1963: Paul Samuelson* and Eugene Fama (EQ)
 - » Efficient market hypothesis
- 1964: Bill Sharpe*/ 1965: John Lintner/ 1966: Mossin (EQ)
 - » Capital Asset Price Model
- 1970: Eugene Fama (EQ)
 - » Développement structuré du concept d'efficience

(j)



Nobel prizes in...financial economics

While the "greeks" talk of academicians, starting in 1970, makes practitioners smile, the list of Nobel prizes grows:

- 1970: Paul Samuelson
- 1972: Hicks & Arrow
- 1976: Milton Friedman
- 1981: Tobin
- 1983: Debreu
- 1985: Modigliani
- 1990: Markowitz, Miller & Sharpe
- 1997: Merton & Scholes
- 2001: Akerlof, Spence & Stiglitz

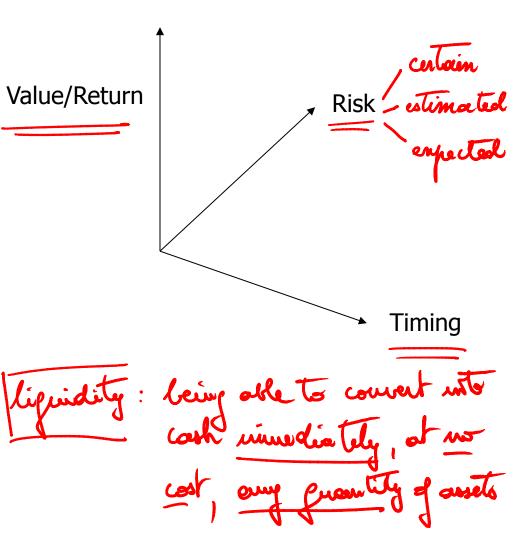


What academicians have been developing?

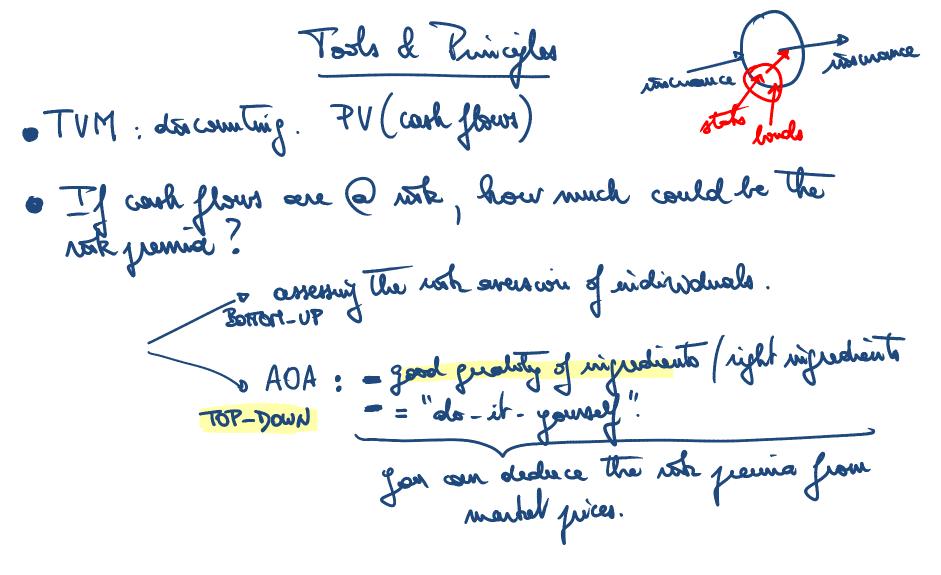
- Dimensions
 - » Value/Return

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- » Risk
- » Timing
- Value/Risk principles
 - » No free-lunch
 - » Market efficiency
 - ✓ Strong form
 - ✓ Semi-strong
 - 🗸 Weak
 - Necessary conditions ?

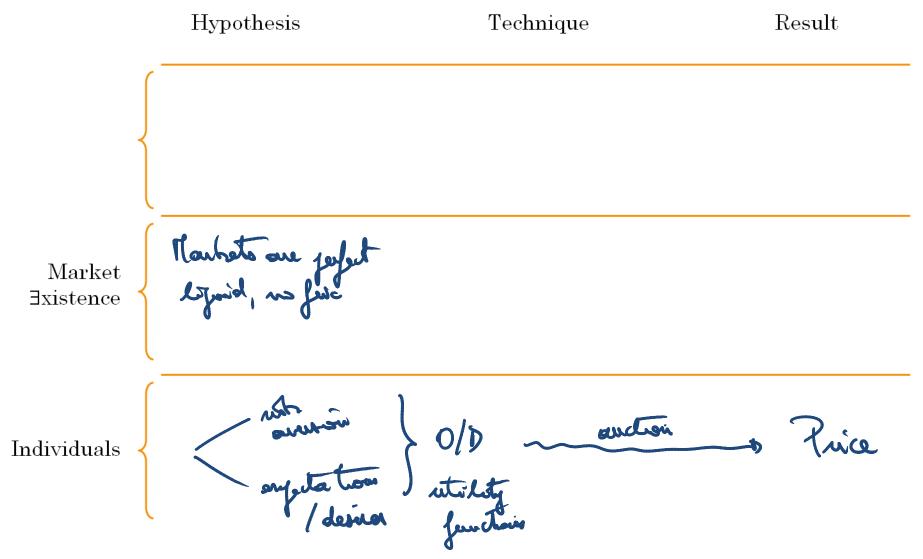








The result: the 3-layers scheme and the current paradigms





The cost of uncertainty



- 2 possibilities to discount a risky cash flow \widetilde{C}_1 occurring in 1 year:
 - » Discount the risky cash flow at a risky rate, i.e. a rate accounting for the cost of uncertainty

Project
$$V_0 = \frac{E(C_1)}{1+k}$$
 where $k = r_f + risk$ premium

» Discount a "certainty equivalent" by a riskfree rate $V_0 = \frac{E(\tilde{C}_1) - z}{1 + r_f} = \frac{E(\tilde{C}_1)}{1 + r_f}$



Example: looking for the risk premia

- All is about the price of risk...how much are you willing to pay for some traded financial asset, its expectation or less than that?
- Let's put into place some framework to examine the problem...
 - » You observe the following endowments (assets & their prices) :

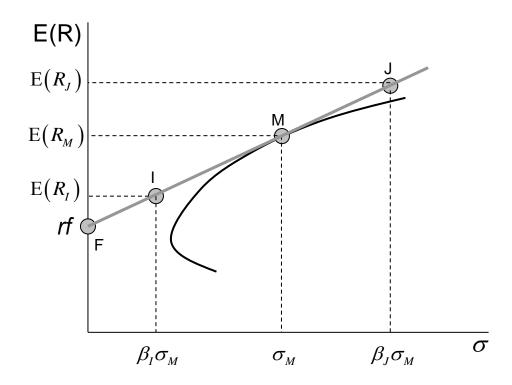
		t=1		
	t=0	Boom up market (<i>u</i>) Proba = 40%	Recession down market (<i>d</i>) Proba = 60%	Expected return
Asset 1 (Bond)	1.00	1.05	1.05	5%
Asset 2 (Market pf)	1.00	2	0.5	10%
Asset 3 (Some project)	?	3	5	
Asset 4	?	0	1	
Asset 5	?	1	0	

» What is the value of the other assets?

Contingent arrets / Derivatives = redundont assets



Some reminder on the CAPM



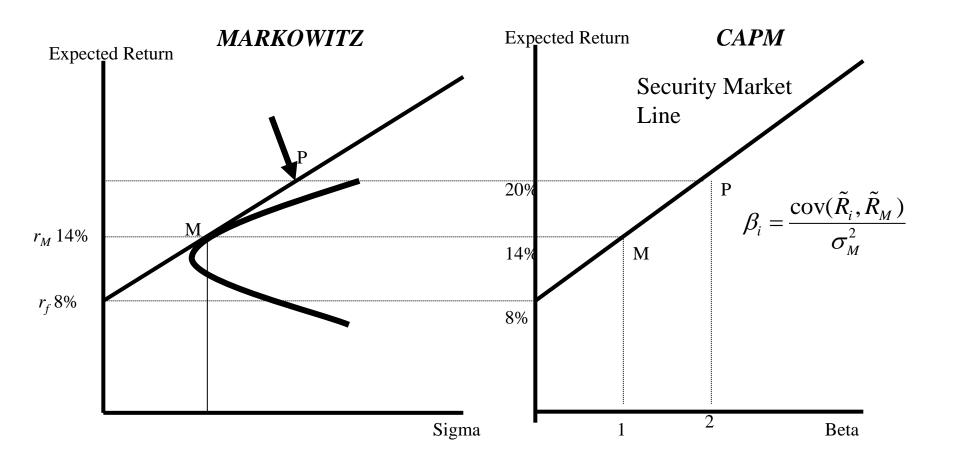
Step 1 $\min_{\forall w_i} \operatorname{var}_{pf} = \mathbf{w} \mathbf{\Sigma} \mathbf{w}$ s.t. $\operatorname{E}(R_{pf}) = x$

How?

- Excel solver
- Elton-Gruber method



From Markowitz to CAPM $k = E(R_i) = r_f + (E(R_M) - r_f)\beta_i$



Layer 1: the CAPM

Consider a future uncertain cash flow *C* to be received in 1 year. PV calculation based on CAPM:

$$V = \frac{E(C)}{1 + r_f + (R_M - r_f)\beta} \quad \text{with:} \quad \beta = \frac{\operatorname{cov}(R, R_M)}{\sigma_M^2}$$

Here: $R = \frac{C - V}{V} \Rightarrow V = \frac{E(C)}{1 + r_f + (R_M - r_f)\frac{\operatorname{cov}(C, R_M)}{V\sigma_M^2}} \quad \text{Define:} \quad \lambda = \frac{R_M - r_f}{\sigma_M^2}$
$$\Rightarrow V \left(1 + r_f + \lambda \frac{\operatorname{cov}(C, R_M)}{V} \right) = E(C)$$

$$V = \frac{E(C) - \lambda \operatorname{cov}(C, R_M)}{1 + r_f} = \frac{\operatorname{Certainty equivalent}}{1 + r_f}$$

See Brealey and Myers Chap 9



Layer 1: Solving...

- Step 1: let's compute some statistics for the market portfolio
 - » For the market portfolio, we know that

	Up-state (u) p = 40%	Down-state (d) 1-p = 40%	
Return	00%	-50%	
Expected return	$r_M = (0.40)(1) + (0.60)(50) = 10\%$		
Market risk premium	$r_M - r_F = 10\% - 5\% = 5\%$		
Variance	$\sigma_M^2 = (0.40)(1)^2 + (0.60)(50)^2 - (.10)^2 = 0.54$		
Price of covariance	$\lambda = \frac{r_M - r_F}{\sigma_M^2} = \frac{.05}{.54} = 0.0926$		

» Reminder

$$V = \frac{E(C) - \lambda \operatorname{cov}(C, r_M)}{1 + r_f} = \frac{\text{Certainty equivalent}}{1 + r_f}$$

$$\operatorname{var}(x) = E(x^{2}) - E(x)^{2} = E\left[\left(x - E(x)\right)^{2}\right]$$
$$\operatorname{cov}(x, y) = E(xy) - E(x)E(y) = E\left[\left(x - E(x)\right)\left(y - E(y)\right)\right]$$



Layer 1: Solving... (2)

Step 2: let's do it for asset 3 (the project)

	Up-state (u) p = 40%	Down-state (d) 1-p = 40%	
Cash flow	3	5	
Expected cash flow	$E(C) = 0.40 \times 3 + 0.60 \times 5 = 4.2$		
Covariance with the market portfolio	$\operatorname{cov}(C, r_M) = (0.40)(3 \times 1) + (0.60)(5 \times (5)) - (4.2)(0.10)$ = -0.72		

Step 3: value the project

$$V = \frac{E(C) - \lambda \operatorname{cov}(C, r_M)}{1 + r_F} = \frac{4.2 - (0.0926)(-0.72)}{1.05}$$
$$= \frac{4.27}{1.05} \underbrace{4.06}$$

we have everything we need to valuate anything...



Layer 1: Solving... (3)

Deduction: the beta

	Up-state (u) p = 40%	Down-state (d) 1-p = 40%	
Cash flow	3	5	
Returns	-26.17%	23.05%	
Expected return	r = (0.40)(-0.2617) + (0.60)(0.2305) = 3.36%		
Beta	$\beta = \frac{r - r_F}{r_M - r_F} = -0.33$		

✓ we have everything we need to valuate anything...



Layer 2: synthetic asset and AOA

- First, we have to assume that the rules of perfect markets and the AOA apply
 - » Relative pricing: if the market is complete, any new asset could be just simply replicated by combining existing assets.
- In our case, this is equivalent to look for the solution of the following system of equations:

$$n_1 \times 1.05 + n_2 \times 2.00 = 3$$

$$n_1 \times 1.05 + n_2 \times 0.50 = 5$$

- The solution is: $n_1 = 5.40$ and $n_2 = -1.33$
- Therefore, the value today of asset 3, our new asset should be, by AOA or even the simple no-free lunch rules depicted before:

$$V_{asset 3} = 5.40 \times 1 + (-1.33) \times 1 = 4.06$$

Otherwise, do it yourself and arbitrage!



Layer 3: state prices = digital options

- Assets 4 and 5 provide us with 1 in one state and 0 elsewhere...
 (They are called Arrow-Debreu securities or contingent claims...)
- If we follow our last discussion, these are redundant assets and their price should also be found easily by replication...
- Globally our problem turns out to be this time :

		<i>t</i> =1	
	Value (<i>t</i> =0)	Up-state (<i>u</i>)	Down-state (d)
<i>u</i> -option	v_u	1	0
<i>d</i> -option	v_d	0	1

Using the same trick as before:

$$n_1 \times 1.05 + n_2 \times 2.00 = 1$$

 $n_1 \times 1.05 + n_2 \times 0.50 = 0$
 $\rightarrow n_1 = -0.32, \quad n_2 = 0.67$
 $\rightarrow v_u = 0.3492$

$$n_1 \times 1.05 + n_2 \times 2.00 = 0$$

 $n_1 \times 1.05 + n_2 \times 0.50 = 1$
 $\rightarrow n_1 = 1.27, \quad n_2 = -0.67$
 $\rightarrow v_d = 0.6032$



Valuation with state prices

 If all state prices are known (markets are complete), then knowing the payoffs in all states allow us to price any asset accordingly:

$$value = v_u \times value_u + v_d \times value_d$$

or

$$value = \sum_{\text{all states } i} v_i \times value_i$$

In equilibrium, the price that you pay to receive 1€ in a future state should be the same for all securities

$$1 = v_u \times 1.05 + v_d \times 1.05$$
$$1 = v_u \times 2 + v_d \times 0.50$$

- » Otherwise, there would exist an arbitrage opportunity.
- An arbitrage portfolio is defined as a portfolio:
 - » with a non positive value (you don't pay anything or, even better, you receive money to hold this portfolio)
 - » a positive future value in at least one state, and zero in other states
- The absence of arbitrage is the most fundamental equilibrium condition.



Fundamental Theorem in Finance

$$value = \sum_{\text{all states } i} v_i \times value_i$$

In our case we have that...
$$v_u = \frac{1 - \frac{0.5}{1.05}}{2 - 0.5} = 0.3462$$
 $v_d = \frac{\frac{2}{1.05} - 1}{2 - 0.5} = 0.6032$

For a value and a return of: $V = 0.3492 \times 3 + 0.6032 \times 5 = 4.06$ $E(r) = 0.40 \times \frac{3 - 4.05}{4.05} + 0.60 \times \frac{5 - 4.05}{4.05} = 3.36\%$



A more general formulation

Imagine the following endowments:

	Current Price	Value Boom	Value Recession
Treasury bond	1	$1+r_f$	$1+r_f$
Stock	S	uS	dS
Contingent claim B	v _u	1	0
Contingent claim R	v _d	0	1

 $u > 1 + r_f > d$

Law of unique price:

$$S = v_u u S + v_d dS \qquad \longrightarrow 1 = v_u u + v_d d$$
$$1 = v_u (1 + r_f) + v_d (1 + r_f) \qquad \longrightarrow v \equiv \frac{1}{(1 + r_f)} \equiv v_u + v_d$$

Solving for v_u and v_d:

$$v_{u} = \frac{1 - \frac{d}{1 + r_{f}}}{u - d} \qquad v_{d} = \frac{\frac{u}{1 + r_{f}} - 1}{u - d} \qquad v_{u} \times uS + v_{d} \times dS = \frac{uS - \frac{udS}{1 + r_{f}}}{u - d} + \frac{\frac{udS}{1 + r_{f}} - dS}{u - d} = S$$



Obtaining "risk-neutral probabilities"

Let's take back each state price:

$$v_u = \frac{1 - \frac{d}{1 + r_f}}{u - d}$$

$$w_d = \frac{\frac{u}{1+r_f} - 1}{u-d}$$

Isolating the discount factor leads to:

$$v_{u} = \frac{\frac{1+r_{f}-d}{1+r_{f}}}{u-d} = \frac{1}{1+r_{f}} \frac{(1+r_{f})-d}{u-d}$$
$$= \frac{1}{1+r_{f}} p_{u}$$

$$v_{d} = \frac{\frac{u - (1 + r_{f})}{1 + r_{f}}}{u - d} = \frac{1}{1 + r_{f}} \frac{u - (1 + r_{f})}{u - d}$$
$$= \frac{1}{1 + r_{f}} p_{d}$$

By definition:

$$v_u + v_d = \frac{1}{1 + r_f} \rightarrow p_u + p_d = 1$$

- And if $d < (1+r_f) < u$ then $0 \le p_u, p_d \le 1$
- These p's are probabilities!!!



"Risk-neutral" pricing

Certainty equivalents can be therefore obtained by using these probabilities

$$C_0 = v_u C_u + v_d C_d = \frac{p_u C_u + p_d C_d}{1 + r_f}$$
Remark
$$p_u C_u + p_d C_d = E(C) - \lambda \operatorname{cov}(C, r_M)$$

1

In our example

$$C_0 = 0.3492 \times 3 + 0.6032 \times 5 = \frac{36.6\% \times 3 + 63.3\% \times 5}{1 + 5\%} = \frac{4.26}{1.05} = \frac{E^c(\tilde{C}_1)}{1 + r_f} = 4.06$$

- *p_u* and *p_d* are **risk-neutral probabilities** such that the expected return, using these probabilities, is equal to the risk-free rate.
- Check

» For the stock
$$p_u u + p_d d = \frac{1 + r_f - d}{u - d}u + \frac{u - (1 + r_f)}{u - d}d = 1 + r_f$$

» For any security $p_u \frac{C_u}{C} + p_d \frac{C_d}{C} = 1 + r_f$



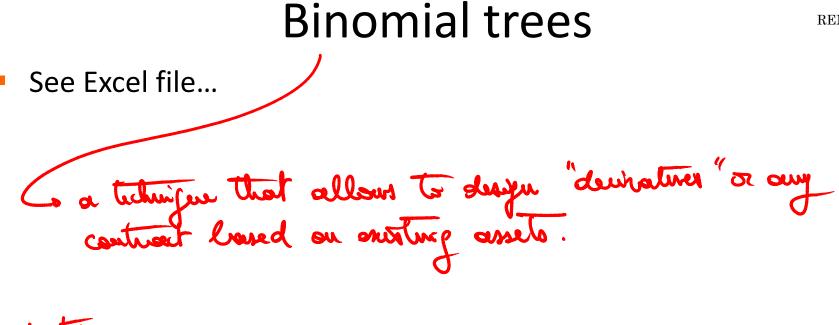
Real World

- In the real world, we have a continuous space of values for the assets as well as a huge number of potential assets to trade for.
- In that probability space, it is not practical to assign a probability to each value that some asset may take. Rather, in a continuous setting, probability becomes a probability density function relying on some main parameter(s). In the case of a normal density function, the variance is the representative of that probability.
- And, if we compare the continuous evolution of those assets, we may be interested in their covariance.
- Now, there is a possibility to relate both worlds:
 - » a discrete one a little bit more complex than the one before, with various time steps and the possibility to reach a wider number of states
 - » And a continuous one where that width is expressed by the variance (or the volatility)
- By using binomial trees, with $u = e^{\sigma\sqrt{T}}$ and $d = e^{-\sigma\sqrt{T}}$ we have it. (this will be explained more in detail in a next session)









1 step

2 A



blass - Steagall Act.

CREDIT RISK



What regulators have been developing?

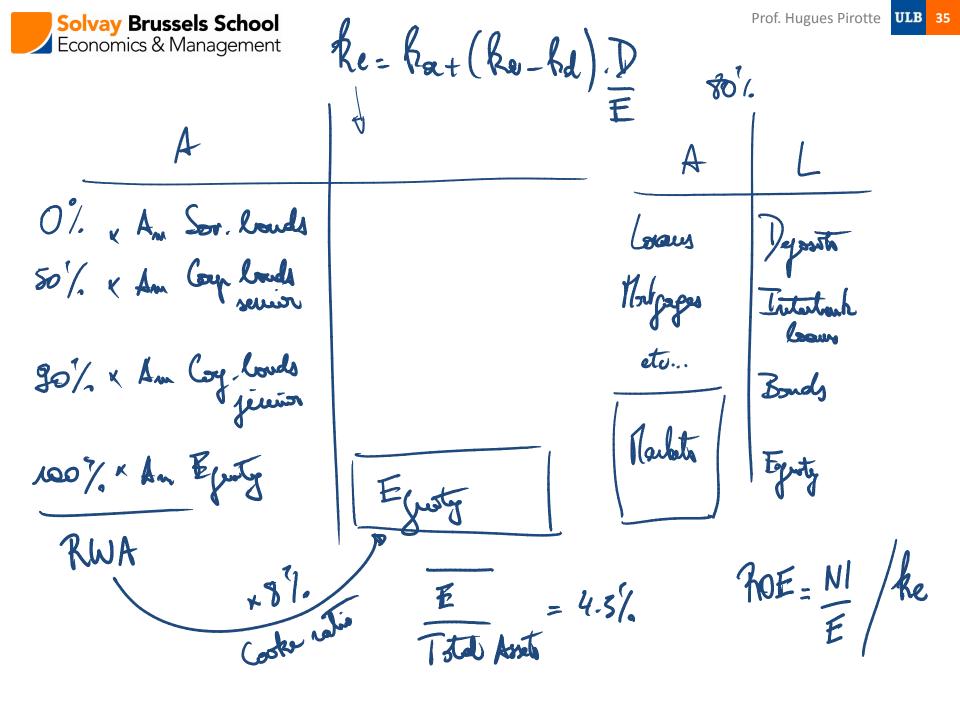
1. Banking crisis of 1930

bank runs, illiquidity of assets, contraction of credit, cash-flow insolvency

Deposit Insurance (FDIC) in 1933

ightarrow US savings & loans crisis in the 80's

- Need for regulation! Why? (Moral Hazard/Adverse Selection/Agency conflicts)
- 4. Herstatt risk
- 5. Decision to create the Basel Committee on Banking Supervision
 - Central bankers from the G-10
 - Safety and soundness of worldwide financial system
 - Common level playing field
 - For core institutions, internationally active
 - Basel Accord \rightarrow recommendations, not legally binding







What regulators ...? (Basel Accord)

- To be implemented by national authorities
 - US: Fed+OCC+FDIC+SEC **>>**
 - UK: Financial Services Authority (FSA) **>>**
 - Japan: Financial Services Agency (FSA) + Bank of Japan »
 - EU: Solvency Ratio Directive (1989) + Capital Adequacy Directive (CAD 1993, **>>** adapted in 1998 to allow use of internal models)
- Sets minimum risk-based levels of capital
- Steps:
 - The 1988 Accord (for credit risk only)
 - The 1996 Amendment (1997, adding market risk): **>>**
 - ✓ allow use of internal models
 - trading book separated from banking book
 - The New Basel Accord (from 1999 to 2004, extensions + operational risk)
 - Three pillars: minimum cap. requirements + supervisory review process + market (1) discipline

{ rule - bressel



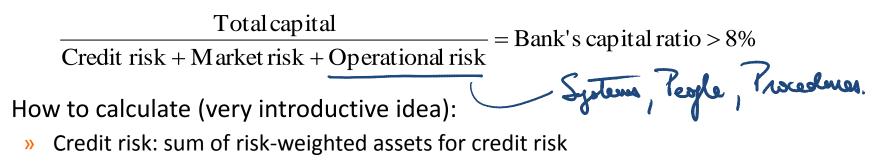




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What regulators...? (Basel II)

Capital adequacy measure



- » Market and operational: multiplication of the MRC and ORC by (1/8%)=12.5
- » Example: bank with €875 in risk-weighted assets and MRC=€20 and ORC=€10, then

 $€875 + [(€20 + €10) \times 12.5] = €1'250$ → 8% × €1'250 = €100

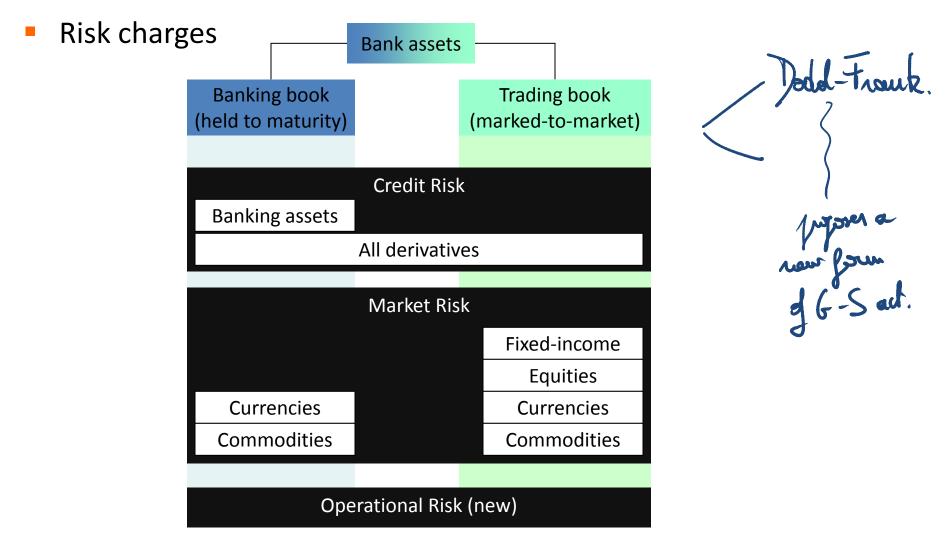
what is equivalent to:

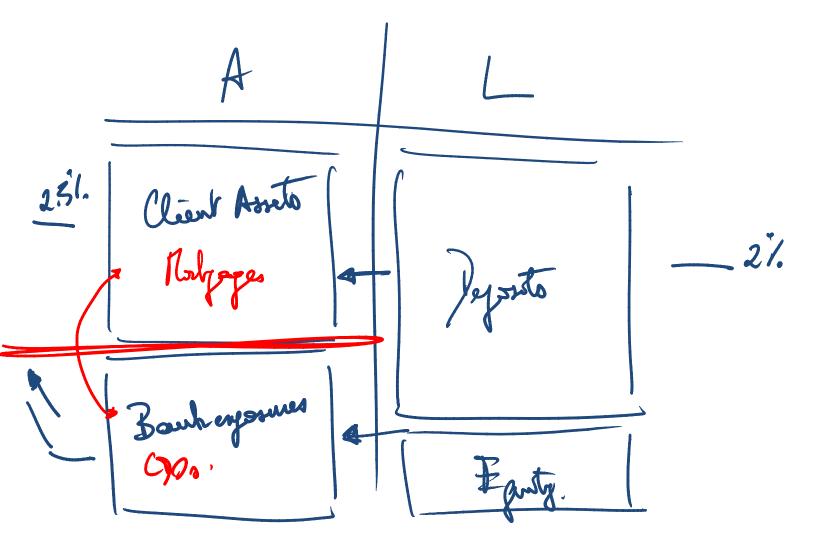
 $8\% \times \in 875 + \in 20 + \in 10 = \in 100$





What regulators...? (Basel II)









What regulators...? (Basel II)

Approaches to measure risk

- » Credit risk
 - ✓ Standardized approach
 - ✓ Internal Rating-based approach
 - Foundation
 - Advanced
- » Market risk
 - ✓ Standardized approach
 - Internal models approach
- » Operational risk
 - ✓ Basic indicator approach
 - ✓ Standardized approach
 - Advanced measurement approach



Forthcoming changes after the financial crisis

- Many forums after the financial crisis...on: liquidity, procyclicality, systemic risk and contribution to it, ...
- Basle III

More focus on supervision and disclosure SFR - Stoble Fundary Ratio (m EU

- » Monitoring of "high cycles" with a new GDP ratio that would make governments oblige their banks to increase the buffer in times "identified" as sumptuous.
- » Numerous news institutions
- IASB

- » More disclosure and separation into risk categories.
- » From IAS 39 to IFRS 9.

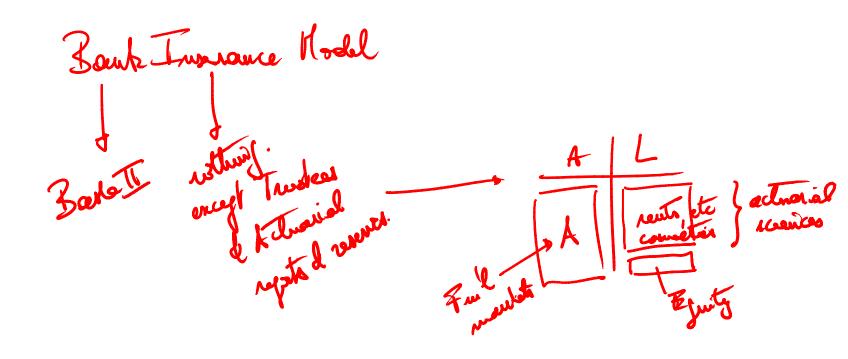




Other regulations

- Insurance
 - » Dutch Solvency Test (DST)
 - » Swiss Solvency Test (SST)
 - » Solvency II (EU) y. Jan. 2015.

» ...







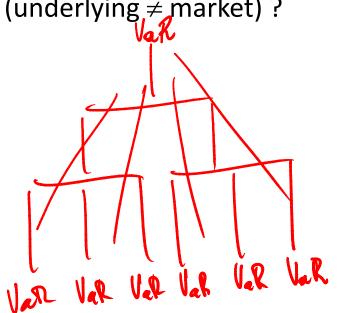
Is Risk Management therefore a new science?

Not really...some examples

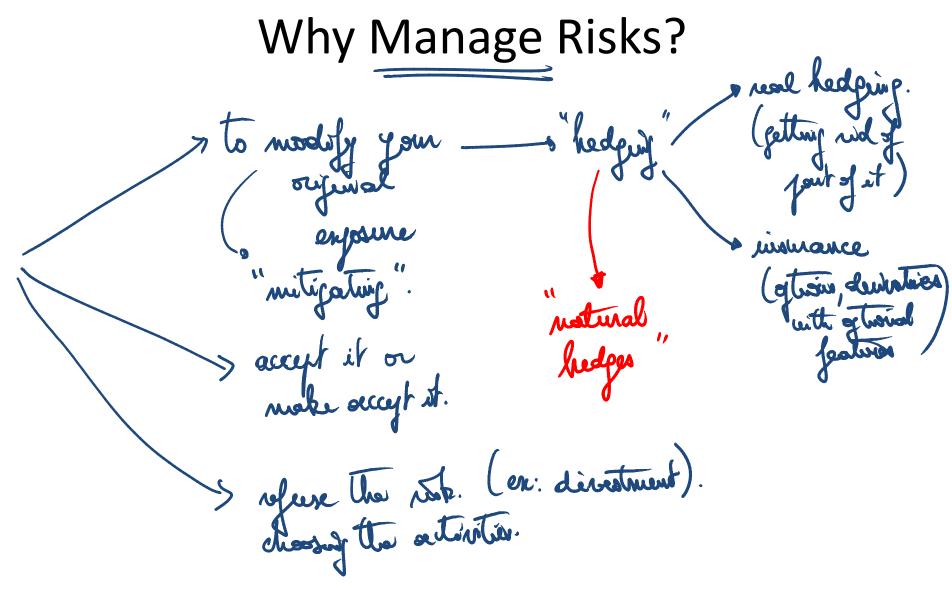


Risk types or sources...

- Market risks
 - » Interest-rates 4
 - » Forex
 - » Stock Market
 - » Commodities
 - » Precious metals
- Business/Activity risks: Weather risk (underlying ≠ market) ?
- Credit risk
- Catastrophe risk
- Operational risk
- Liquidity risk
- Model risk
 - » Biased modelling
 - » Bad implementations











The scope of risk management...

- Risk Monitoring
 - » Identify
 - » Price/Quantify risk...
 - ✓ VaR...and other measures
 - 🗸 CaR
- Risk Management Objectives
 - » Avoid
 - » Retain
 - » Mitigate/Control/Transfer:
 - ✓ Diversify *V*
 - Immunise
 - 🗸 Hedge 🗸
 - Naturally (REAL ASSET SIDE)
 - By synthetisation (SHARING CONTRACTS)
 - ✓ Protect/Insure
 - With products/contracts (RIGHTS)
 - With collateral (EXCHANGE)